FRACTALS AS THE MAIN COURSE, NOT THE GARNISH

Students find fractals fascinating. Normally, fractals is shown to be a "cool" application to the concept just learned. What if a unit was organized around fractals, and the math was then organized around understating this interesting phenomenon?

This article is about organizing the math around fractals, as opposed to around the math topics. Fractals spark students creativity and curiosity while making connections to art, nature and science. Fractals can lead into many areas of math content, this article will focus on: area, perimeter, exponents, sequences and series, logarithms and complex numbers.

WHAT IS A FRACTAL

Our first exploration is to grapple with a similar problem to that of Benoit Mandelbrot when he was working with fractals. We asked the students to find the perimeter of the continental United States. Upon discussion this brought out the inconsistency when measuring irregular perimeters. How the scale of the map affected the solution. The larger the map, the longer the perimeter the students got.

We then set out to find what a fractal is and what are it's uses. The students get to be researchers and our main tool was the internet (which has much higher quality pictures than I could draw). So internet was a key tool in discovering what is a fractal They are trying to figure out what this intriguing pattern is and how does it apply to the world. Also, the students came up with a definition of fractals in their own words. One class agreed on " an object that looks similar at any level of magnification." The internet also has interactive activities to explore fractals and see where fractals are in the natural world.

Another benefit to working with fractals is students are working in a relatively new field in math, and can feel like part of the cutting edge in mathematics. A student said, *"It's brave to look at a not completely defined math in high school. This is exploring something you can't control. It's not some structured thing to hand down from teacher to the student."*

Once the class comes to agreement on a definition of a fractal, we started looking at a few specific fractals. We looked at the Tree Fractal, Box Fractal and Koch Snowflake (see figure 1, also see Mathematics Teacher April 1999 for worksheets). Here students worked on measuring length, perimeter, area, summation notation and representing the growth for each iteration with exponential functions. The general formulas for the Box Fractal are:

- side length is $(1/3)^n$
- number of squares is 5^n
- number of sides is $4(5)^n$
- perimeter is $4(5/3)^n$, where n is the iteration

Some good questions for discussion are: What happens to the perimeter as the number of iterations goes towards infinity? What happens to the area as the number of iterations goes towards infinity?

Similarly, this can be done for the Koch Snowflake, though there is some extra challenges. One in finding the area of the Koch Snowflake, the students need to find the area of equilateral triangles. Also, a way of finding what is happening with the area can lead into infinite geometric series (see figure 2).

The general formulas for the Koch Snowflake are:
•side length is $(1/3)^n$
•number of sides is $3(4)^n$
•perimeter is $3(4/3)^n$, where n is the iteration

DIMENSION

Usually when we talk about dimensions, we think of topological dimension, where the dimensions are usually whole number values for a point, line, plane of 0, 1, 2 and 3 being the dimension we live in. Though fractals don't fit nicely into that dimensional system and have their own dimensional system.

We know that if you double the side length of a cube, the perimeter will double, the surface area will quadruple and the volume will be eight times as large (see table 3).

Though if we look at a fractal that may not hold. If we look at the Sierpinski Triangle (see figure 4), remember the white is the actual area, the black are void and doesn't count as area, then the dimensional rule doesn't hold. If we double the side length, there is three times the area. How does this fit then with our model of dimensions?

One way to start exploring the idea of dimensions different from what we're used to is by reading and discussing the book *Flatland,* by Edward Abbott. This looks at how a two dimensional life would perceive a third dimension object. We then discuss what would a fourth or a fractional be like?

For calculating the fractal dimension, like that of the Sierpinski Triangle, of an object we use the formula $N=1/r^D$.

D = fractal dimension
N = number of elements or number of equal pieces
r = size of ruler or amount scaled down by

To calculate the fractal dimension this can be written $D = \log N/\log(1/r) = \log N/ -\log(r)$, since $\log(1/r) = -\log r$. Hence, the lead into logarithms to be able to calculate dimension. Taking the Sierpinski Triangle the number of elements is 3, and it is scaled down by a factor of $r = 1/2$. This gives us a fractal dimension of $D = \log 3 / -\log (1/2) = 1.585...$ That still begs the question for discussion of what does a dimension of 1.585 mean in the physical world?

GRAPHING FRACTALS

We started with having students trying to sketch fractals in nature and fractals of their own design. Also, we critiqued their works and some of the famous fractals to look at why fractals are so esthetically pleasing. We also tried making fractals with paints and clay. The main obstacle was how to create the level of detail necessary to create the self-similiarity?

This led into the iterative equations and how fractals are graphed. The equations are simple, $z = z^2 + c$, where c is some constant. Though then we looked at how the process of plugging the result of the equation effected the number. Complex numbers are used, with the imaginary component as the y coordinate, and the real component was the x value. Therefore leading into operations with complex numbers (See figure 5).

There are many more possibilities than the few I've listed here. What was central though was that the math introduced in the class was all in order to understand fractals and their representation of the world.

REFERENCES

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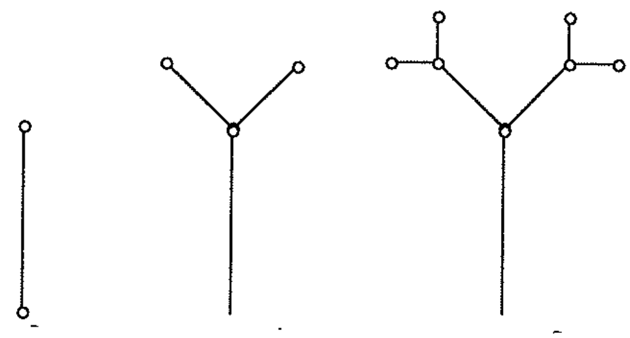
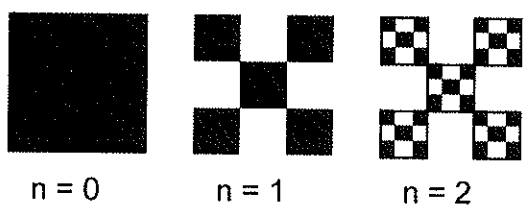
Lauwerier, Hans. *Fractals*. Princeton University Press, 1991.

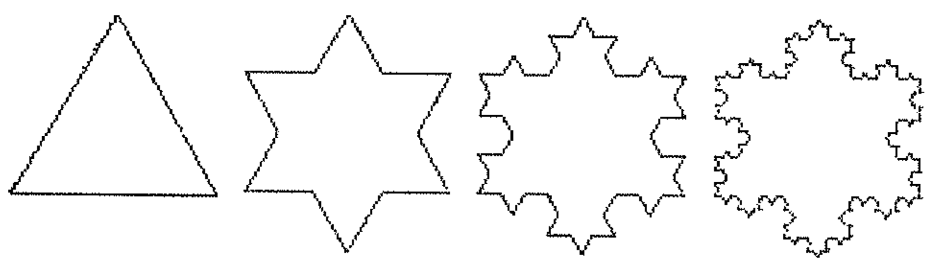
Naylor, Michael. "Exploring Fractals in the Classroom." *Mathematics Teacher* 92 (April 1999): 360 - 67.

FRACTAL ARTICLE FIGURES AND TABLES



BOX FRACTAL





Traditional Dimensions

Parameter	Dimension	Increase
perimiter	1	2 = 2^1
area	2	4 = 2^2
volume	3	8 = 2^3

Figure 3

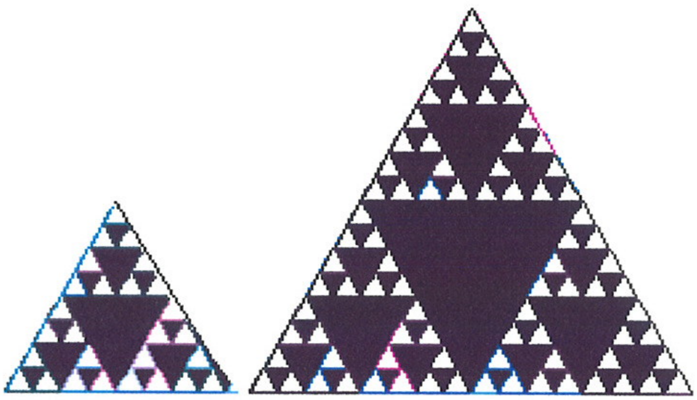


figure 4

Iterations of $z = z^2 + (1 + i)$

(seed) n = 0	2 + 3i	1 + 3i
n = 1	-4 + 13i	-7 + 7i
n = 2	-152 - 103i	1 - 97i
n = 3	12496 + 31313i	-9407 - 193i